MTH 516/616: Topology II

Homework II

(Due 03/02)

1. If A is a retract of X, then the homomorphism $j_* : H_n(A) \to H_n(X)$ induced by the inclusion $j : A \hookrightarrow X$ is injective.

2. Verify that if $f \simeq g$, then $f_* = g_*$ as induced homomorphisms on reduced homology.

3. (a) Show that $H_0(X, A) = 0$ iff A meets each path-component of X.

   (b) Show that $H_1(X, A) = 0$ iff $H_1(A) \to H_1(X)$ is surjective, and each path-component of X contains at most one path-component of A.

4. If SX is suspension of X, then show that $\widetilde{H}_n(SX) \cong \widetilde{H}_n(X)$. [Hint: SX is the quotient space obtained from taking two copies of CX and then identifying their bases.]

5. Compute the homology groups $H_n(S^2, A)$, where A is a set two distinct points in S^2. What if A has n distinct points?

6. **(For practice)**

   (a) Derive the long exact sequence of reduced homology groups of a pair of spaces (X, A).

   (b) Derive the long exact sequence of homology groups of a triple of spaces (X, A, B), where $B \subset A \subset X$.